

**S. S. Leshchenko** (AESC MSU). **About one type of barycenters for a finite set of probability measures.**

Let  $\mathbb{E} = (\Omega, \mathcal{F}, (P_\theta)_{\theta \in \Theta})$ ,  $\Theta = \{1, 2, \dots, N\}$  be a statistical experiment. Let  $R$  be a probability measure on  $(\Omega, \mathcal{F})$  that dominates  $P_1, \dots, P_N$  and  $p_1, \dots, p_N$  — corresponding Radon-Nikodym derivatives.

Define a Markov kernel  $\mathbf{P}$  from  $(\Theta, 2^\Theta)$  to  $(\Omega, \mathcal{F})$  by  $\mathbf{P}(\theta, A) = P_\theta(A)$ ,  $\theta \in \Theta$ ,  $A \in \mathcal{F}$ . Let  $\mathcal{K}$  be the class of all convex lower semicontinuous functions  $f : \mathbb{R} \rightarrow \mathbb{R} \cup \{+\infty\}$  such that  $\text{int dom } f = (0, +\infty)$ .

Let  $\pi$  and  $\bar{\pi}$  be two probability measures on  $\Theta$ . Suppose that  $\pi_i = \pi(\{i\}) > 0$  и  $\bar{\pi}_i = \bar{\pi}(\{i\}) > 0$ ,  $i = 1, 2, \dots, N$ . For the statistical experiment  $\mathbb{E}$ , a measure of informativity is proposed, constructed using  $f$ -divergence ( $f \in \mathcal{K}$ ):

$$\mathcal{I}_{f, \pi, \bar{\pi}}(P_1, \dots, P_N) = \inf_{\mathbf{Q} \in \mathcal{P}(\mathcal{F})} I_f(\pi \times \mathbf{P}, \bar{\pi} \times \mathbf{Q}). \quad (1)$$

Here  $\mathcal{P}(\mathcal{F})$  is the set of all probability measures.

The solution of the problem (1) will be called the barycenter. We will be interested in the question of the existence of a barycenter. Using convex analysis methods, we formulate a dual problem and obtain a characterization of the barycenter through the solution of the dual problem. Let us formulate the main result.

**Theorem.** *The barycenter in the problem (1) exists and the dual problem has a solution. In addition, there is no duality gap and there is a characterization of the barycenter of the problem (1) through the solution of the dual problem.*

Note that if the distributions of  $\pi$  and  $\bar{\pi}$  coincide, then the existence of a barycenter was proven in [1] by other methods, and the characterization of the barycenter of the problem (1) coincides with the characterization obtained in [1].

#### BIBLIOGRAPHY

1. I. Csiszár. A class of measures of informativity of observation channels. *Period. math. Hungar.*, 2:191–213, 1972.